Method of Determining Visibility From a Contoured Map. . . . . .

Lecture Delivered Before the
Infantry and Cavalry Class.

OCTOBER, 1906

Method of Determining Visibility From a Contoured Map.

Rules for determining the visibility of one point of ground from another by means of a contoured map are given in many military text books treating of maps, but the methods there given are generally laborous and time consuming. For instance, one book advises its students to construct a profile of the ground in the same vertical plane as the line of sight, and to solve the problem from the profile. Such a method will give accurate results, but when applied to any considerable area it becomes prohibitive, and if questions of visibility could be answered only by such cumbersome solutions, maps would have but little value for this purpose on account of the amount of work involved.

As the value of a map increases in proportion to the uses to which it can be put, it follows that any method which enables questions of visibility to be answered quickly and with relatively little work, will enhance its usefulness to a corresponding degree. It has seemed advisable; therefore, to explain that method which is believed to be the most convenient.

It should be remembered, however, that owing to the irregularity of the ground in most localities such problems often become complex even when solved by the best methods, and that none has yet been devised which will answer such question without some work.

In obtaining the answers to such questions the curvature of the earth is not ordinarily taken into consideration, owing to the short distance at which

animate objects are visible. The earth curves away from its tangent at a rate equal, in inches, to the square of the number of miles multiplied by eight. Thus at one mile the surface is 8 inches below the tangent, at two miles it is $2 \times 2 \times 8 = 32$ inches; at three miles it is $3 \times 3 \times 8 = 72$ inches.

The line of sight, however, does not coincide with the tangent, but is itself a curved line, concave towards the earth, owing to the refraction of the rays of light. Light moves in a straight line in a homogeneous medium, but undergoes a deviation in passing from one medium into another of different density. In passing from a lighter to a denser medium the ray of light is bent towards the perpendicular to the surface limiting the media, in passing from a denser to a lighter medium it is bent from the perpendicular. If the medium varies uniformly in density the bending becomes regular and the ray of light follows a curve. Owing to the fact that the air is denser near the surface of the ground, the ray of light is curved gradually downwards and appears to come from a higher point than it actually does, thus counteracting the curvature of the earth by an amount equal to the refraction. This varies with the state of the atmosphere but its average value for objects upon the earth's surface may be taken as 1.3 inches times the the square of the number of miles.

The separation in inches between the line of sight and earth's surface is, therefore, the square of the distance in miles times 6.7. For one mile it is 6.7 inches; at two miles it is $2 \times 2 \times 6.7 = 26.8$ inches; at three miles it is $3 \times 3 \times 6.7 = 60.3$ inches; at three and a half miles it is $3.5 \times 3.5 \times 6.7 = 82$ inches. The observer's eye is ordinarily five feet or more above the ground; therefore, if, in questions of visibility, at distances of more than three miles, the point of sight be taken at the surface of the ground,

the answer obtained should be correct for the slightly greater distances at which troops can be seen with the ordinary field glasses. Many officers prefer to take the point of sight at the surface of the ground for all except very short distances, as in this manner the discrepency is on the side of safe solution, and tends to counteract the possible slight increase of elevation on intervening ground due to growth of vegetation.

For distances, then, at which visibility problems would ordinarily be solved from the map for military purposes, the curvature of the earth may be neglected, and the problems more easily and completely solved by one plane descriptive geometry, than by any other method.

In one plane descriptive geometry an object is represented by its projection upon a horizontal plane, and by numbers upon this projection showing the height above the plane of such points and lines of the object as may be necessary to determine it. An irregular object, such as the surface of the ground, is shown by projecting the lines cut from it by equidistant horizontal planes, that is by its contours.

The plane upon which the projection is made is termed the plane of projection, the plane of reference or plane of comparison. For maps, it is ordinarily taken as at sea level, where data for determining the height above the sea exists.

The numbers which show the heights of points and lines above the plane of projection, are termed references, and on maps in this country are generally expressed in feet; in countries using the metric system references are most frequently expressed in meters.

The references of single points are generally inclosed in parenthesis and are placed to the right of the point. Thus Fig. 1 shows the references of the

two points, one, a, 36.45 feet, the other, b, 250 feet, above the plane of projection. The reference is generally written so that the projection shows between the parenthesis and the first figure, but when this position, from the nature of the drawing, is not clear or advantageous, other positions may be used, as is shown in the case of the point whose reference is 178.60, Fig. 1.

The position of a straight line is indicated when the references of two of its points are given. Thus the line ab, Fig. 2, is fixed by the position of its points 65.0 and 70.0 If the line c d have the same point 65.0 as a b, the lines must intersect at that point. If the line is horizontal its reference is written along its projection, as shown in Fig. 3. If the references of a line are written at regular intervals, they form a scale of declivity of the line, as shown in Fig. 4.

The slope, or declivity, of a line is measured by the angle which the line makes with the horizontal plane. It may be given in degrees, as a slope of 1°, 2°, etc., or it may be expressed in terms of the natural tangent of the angle. In this latter case the value of the tangent is usually expressed in the form of a common fraction, in which the vertical distance between any two points is the numerator, and the horizontal distance between the same two points is the denominator, as $\frac{1}{1}$ or $\frac{2}{3}$. A slope of 1° is the same as $\frac{1}{57.3}$. Having the references of two points of a straight line, and knowing the scale of the map or drawing, the declivity of the line can be determined. If Fig. 2 be on a scale of 200 feet to 1 inch, and the points whose references are 65.0 and 85.0 on cd are 0.8 inch apart the declivity of cd is $\frac{20}{160}=\frac{1}{8}$. Similarly, the horizontal distance between 65.0 and 70.0 being 0.9 inch, or 180 feet, the slope of ab is $\frac{5}{180}=\frac{1}{36}$. It is evident that if the fraction showing the slopes

of several lines have a common numerator, that line is the steepest in which the denominator is the smallest; and, conversely, that one of the lines has the gentlest slope which has the largest denominator. If the denominators are the same, that line is steepest which has the largest numerator: or in other words the larger the value of the fraction the steeper the line.

If H represent the horizontal distance between two points of a line and V the difference of their elevations, or references, the natural tangent of the angle of the line is $\frac{V}{H}$. For any other pair of points of the line the tangent may be represented by $\frac{V'}{H'}$; but as the slope of thc line is uniform and the angle the same at all points $\frac{V'}{H'} = \frac{V.}{H}$ $V' = H' \times \frac{V}{H}$. $H' = V' \div \frac{V}{H}$. That is, if the slope fraction $\frac{V}{H}$ and horizontal distance, H′, between two points of a line are given, their vertical distance is found by multiplying the horizontal distance by the slope fraction. If the slope fraction and vertical interval be given, the horizontal distance may be found by dividing the vertical distance by the slope fraction.

Occasionally in a military work slopes are given by placing the vertical distance in the denominator and the horizontal distance in the numerator of the slope fraction. Thus a slope of two vertical to three horizontal is written $\frac{3}{2}$. This expreses the slope in terms of the tangent of the angle made by the line with the vertical. Angles are, however, almost invariably measured from the horizontal, rarely from

the vertical, and the method of writing the slope fraction $\frac{V}{H}$ is therefore decidedly preferable to $\frac{H}{V}$.

The problem of determining the visibility of a relatively small object, or point, from a given place may be solved in two ways. First by ascertaining whether the straight line from the object to the eye passes above all the intervening ground; second by finding whether the straight line from the eye and tangent to the intervening ground passes above or below the obiect. Each method involves the determination of the elevation of a third point of the line, two points being given, and circumstances will decide which will be the more convenient.

As an illustration, suppose it is required to ascertain from the map of the Fort Leavenworth reservation, whether the small railroad bridge, or culvert, on the Union Pacific railroad track at A, is visible from the point marked B, elevation 920, at the upper end of the target range. As the ends of the bridge lie practically in the 840 contour, the elevation of the bridge may be taken at that figure. Constructing the scale of declivity of the line AB, it is found that the line lies everywhere above the surface of the ground and the point is visible. By using a scale of equal parts the scale of declivity of the line can be read without the time and work necessary to make the actual construction. Thus, if the map distance from A to B is $\frac{86}{30}$ inches, the 880 reference of the line must be at $\frac{43}{30}$ from A or B, the 860 point of the line will be $\frac{21.5}{30}$ from A, and similarly for any other points.

If the question were the visibility of the bridge from D, reference 870, an inspection of the map shows that the intervening ridge having 860 for its

highest contour is the ground most likely to obstruct the view. Since the line of sight slopes downward, the further side of the hill will be the portion to which the line of sight will be tangent, and the 860 contour at E will be a point of the line, (a line from D, 870, through F, 860, would pass below E, 860, therefore into the hill, and could not be a line of sight to any point behind the hill). Having given D, 860, and E, 860, it is found by measurement of the map that the distance DE=H=$\frac{31}{30}$ inches, the distance EA=H′=$\frac{28}{30}$ inches the vertical distance between D and E is 10 feet hence the vertical distance between E and A, or V′, $=\dfrac{10\times28}{31}=9.0$ ft. The reference of A on the line of sight DE is therefore 860−9=851. The bridge being at 840, cannot be seen from D.

The same method might have been used on the line AB, the contour 860 at C being the point which determines the position of the line of sight. Measuring the map, we have the distances, AC=$\frac{31}{30}$, CB=$\frac{55}{30}$. Substituting in the equation $V'=\dfrac{VH'}{H}$, 60 for V, 55 for H and 31 for H′, we find V′=33.8, or the difference of elevation between A and C is 33.8 ft. A is therefore 860−33.8=826.2, which is below 840, and the bridge is visible. If it were desired to ascertain the elevation of A compared with B, H′ becomes 86 and $V'=\dfrac{60\times86}{55}=93.8$. The reference of A is then 920−93.8=826.2, the same as before.

If the point of sight were at a lower level than the intervening ground, as at A, and it were desired to ascertain the visibility of higher points, as B and D, the solution would be the same. In this case,

however, the nearer branch of the 860 contour would be the determining one; but this is the same branch that was the further when the point of sight was at B and D. That this is the controlling branch when the point of sight is at A can be readily shown by laying off the line, A 840, F 860, which will be found to pass below the 860 contour at E, and hence through the hill.

These solutions have not taken into account the fact that (since the hill has not a perfectly flat top) the ground along the ridge must be higher than 860. This ground above the *highest* contour is ordinarily neglected, for three reasons: first, in close distances the fact that the point of sight has been taken at the surface of the ground is generally sufficient to counter balance any error due to height of intervening ground above the highest contour; second in long distances the line of sight is generally tangent at some other point of the hill than the summit; third, on most maps the contours are not run with such a degree of accuracy as to justify elaborate constructions. In case there is reason to believe that the top of the hill controlling the line of sight, is practically on a level with the point of sight, or where an accurate solution is necessary from a reliable map, it may be advisable to interpolate a contour at less than the full interval.

The bridge A being visible from B and invisible from D, there must be a place between B and D where it changes from one condition to the other. This point can be found tentatively by drawing lines from A so as to touch the 860 contour and constructing their scales of declivity. That one which has the same reference as the corresponding point of the ground along the line BD will be the point desired. A much more rapid solution can however be made mentally with the use of a scale of equal parts. Placing the

zero of any scale, say thirtieths of an inch at A, and starting with the line AD, the distance to the critical contour 860 at E is found to be 28 divisions. The point 870 of the line would therefore be at the distance of 42 divisions from A which is short of D, and by the time the line reaches the vertical through D it must be higher than 870. Swinging the scale to the point where the line BD crosses the 880 contour at L, we find the distance from A to the 860 contour to be 29 divisions, therefore the 880 point of the line would be 58 divisions from A, while the scale shows the 880 contour to be at 72; A is still invisible. Swinging the scale to the 900 contour we find the distance to the 860 contour from A to be 30, the 900 point of the line is therefore at 90 divisions from A, while the scale shows the 900 contour to be at 80 divisions. The line therefore passes below the point K and the bridge is visible. It has become visible between L 880 and K 900. Swinging the scale to the 890 point of the ground along BD, that is to the point midway between K and L, we find A to 860 contour, 29 divisions; A to 890 point of line of sight 72.5; which is practically the distance from A to the point midway between K and L, which is therefore the point at which the bridge begins to be visible in passing from D towards B, or ceases to be visible in going from B to D.

The problem could have been worked by taking the zero of the scale at points on B P and mentally calculating whether the resulting lines of sight passed above, below or through A, but while the solution is the same in result on whichever side of the critical contour the zero of the scale is placed, there is at times considerable difference in ease of working, owing to the numbers involved.

The solutions given were for an isolated object and in a terrain where the critical contour was easily

recognized. By critical contour is meant that one limiting the line of sight in reference to any particular object, or in other words, it is the contour at which a particular line of sight is tangent to the intervening ground. Different lines of sight from the same point of sight may have different critical contours, as is almost certain to be the case when the object whose visibility is to be ascertained, is of any considerable extent, as a stretch of road.

To determine the point of tangency of the line of sight, i. e.: the critical contour, in many cases requires a special construction, but one which is easily and quickly made. If the point of sight is at a lower level than the intervening ground the tangent line of sight in any vertical plane is the steepest line of sight or straight line from the point of sight which touches a contour. Any lines of sight less steep would strike the ground and be interrupted. Conversely, if the point of sight be at a higher level than the intervening ground, the limiting line of sight would be that with the gentlest slope which touches the hill.

If it be required, Fig. 6, to find the line from A, in the direction AB, which shall be tangent to the ground shown by the contours 30 to 60, draw an auxiliary line in a convenient direction, and construct its scale or declivity arbitrarily of convenient dimensions to conform to the portion of the map under consideration. Draw the horizontals, 30, 40, etc. between the contour points along the line of AB and the auxiliary line. The point of tangency will then be on that contour whose horizontal makes the smallest angle with the auxiliary line on the side towards the point of sight. In this case the smallest angle is at D, and the point of tangency is at E on the contour 50. If the scale of the tangent line, that is, the line A10—E50 were desired, it could be easily con-

structed by drawing to it lines parallel to DE from the points 20, 30, etc. of the auxiliary line. That the smallest angle as indicated gives the steepest line, can be seen by drawing from D a line parallel to any of the other horizontals, in this case taken as De, parallel to the horizontal of 60. The angle being larger, the point e must be further than E from A, and hence for e the value of H in $\frac{V}{H}$ must be larger than the H for E; but the smaller the value of H, V remaining the same, the steeper the line.

In, Fig. 7, the point a of the line ab is taken above the intervening ground, and by a similar construction it is found that the point of tangency is at e on the contour 80, the angle ade being the largest between horizontals and auxiliary line towards the point of sight, a, hence giving the line of gentlest slope from a. But if the angle ade is the largest the angle cde must be the smallest; hence the point of tangency is always indicated by the smallest angle towards the *descending* portion of the auxillary line. (The auxiliary ak is used to determine the tangency from b40 of the line of sight.)

In solving a problem it is not necessary that the various horizontals be drawn, as the position of the smallest angle can be easily determined by sliding a triangle along a straight edge, shifting the position of the latter whenever an angle smaller than any previous one is found.

As an illustration of the value of this method, let it be required to determine from the Fort Leavenworth map the visibility of the road A B from the point X. (Fig. 8.) Two auxiliary lines X Y and X Z were used; one line drawn in an easterly direction from X might have been used but would have complicated the drawing somewhat more. The points of

tangency of the different lines of sight are indicated, and from them it is seen that the road is visible from A to C and from D to E and invisible at other portions.

The method can also be used to determine the question of visibility of areas as is shown in Fig. 9, which is also a portion of the map of the Fort Leavenworth reservation and vicinity. Let it be required to ascertain the ground visible from the point X on the 900 contour to the southeast of Sentinel Hill. Two auxiliary lines X Y and X Z are used, the latter being double, as shown by the two sets of references, one for ascending lines of sights, the other for decending. The points of tangency of the lines of sight upon the north and west of Atchison Hill were determined by finding the smallest angle corresponding to each, and the line dividing the visible from the invisible area formed by connecting these points in the proper order. The visible area upon the hill to the west of Atchison and Government Hills was found by determining the points of tangency for the top line, and for the bottom line ascertaining the points where the lines of sight tangent to Atchison Hill prolonged struck the ground. On Sentinel Hill, owing to its shape, its steepness and the fact that there is nothing between it and the point of sight, the points of tangency are practically where the lines from X are tangent to the curves of the contours. The lines of sight tangent to the 880 and the next two or three lower contours on the north side of Sentinel Hill, strike the ground again on the south slope of the hill on which the Taylor house stands, permitting the larger part of that slope to be seen. A portion of the narrow ridge beyond this hill is visible over its top. The spurs on the west of Sheridan Drive present a succession of visible and invisible areas, the upper portions being determined by finding the points of

tangency of lines of sight, the lower portions by finding where the lines of sight tangent to the preceding spur strike the ground. On the spur B it will be noticed that the dividing line is taken as running down the crest of the ridge, and not along the further branches of the contours. This is apt to be the case on ridges in any degree sharp, and construction of one or more lines of sight will often show, as in this case, that the further branch is invisible. A small area about four hundred yards to the northward of the point of sight is invisible owing to a sudden fall in the ground.

The problem was solved from the contours as shown on the map, neglecting the effect of vegetation. Had it been desired to make allowance for the trees, this could be done by increasing the reference of each contour on wooded ground about 30 feet, or perhaps 40 feet.

Fig. 1.

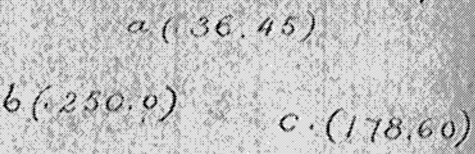


Fig. 2.

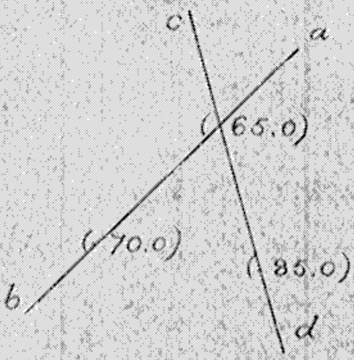


Fig.

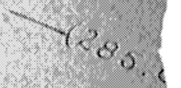


Fig. 4.

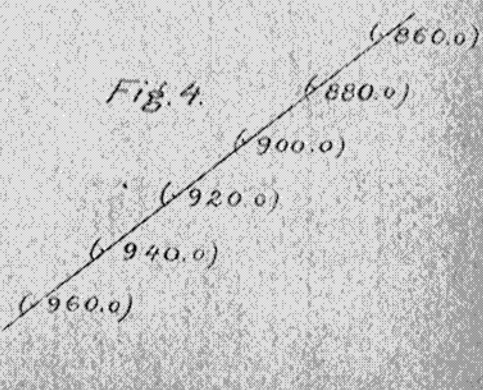


Fig. 5.

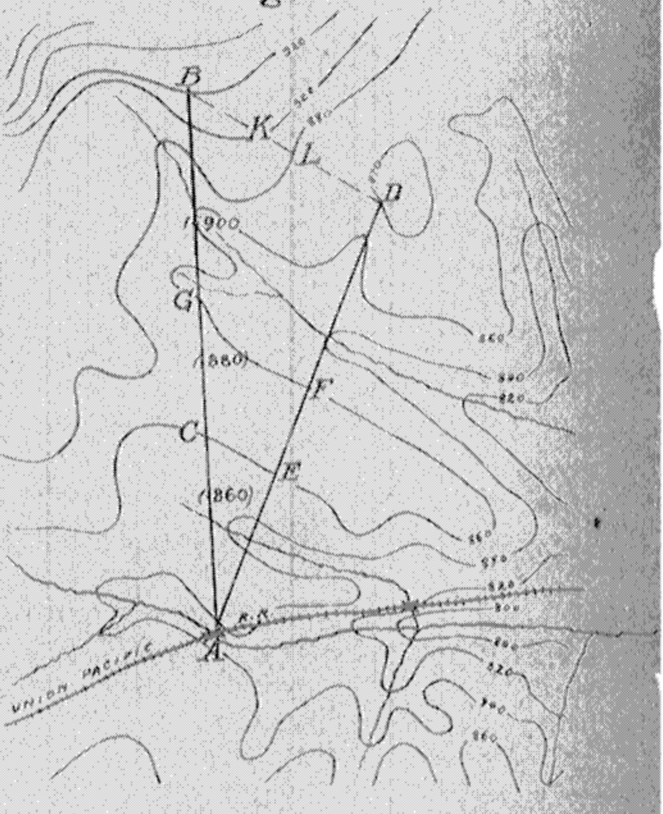
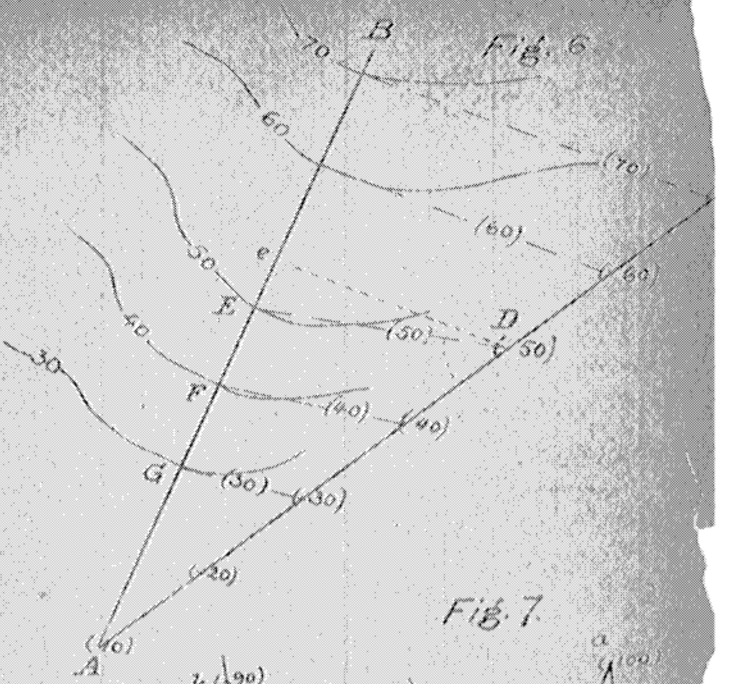
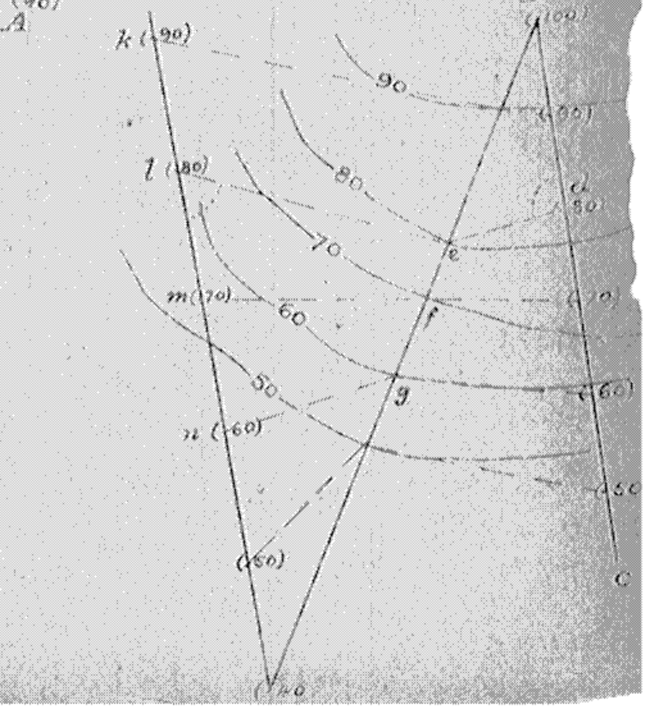


Fig. 6



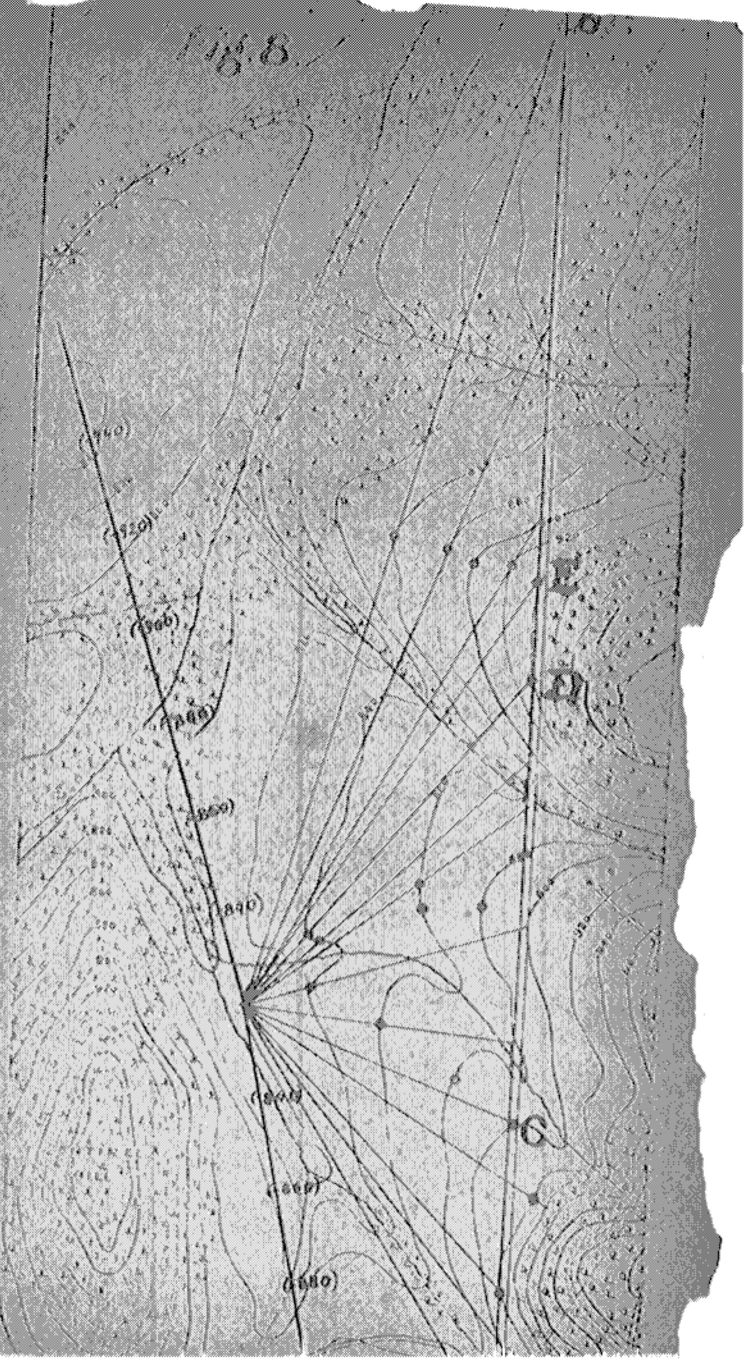
B
70
60
(70)
(60)
50
e
(60)
E
D
(50)
(50)
40
30
F
(40)
(40)
G
(30)
(30)
(20)
(40)
A

Fig. 7



a
k (90)
90
l (80)
80
d
70
e
m (70)
60
f
g
50
(60)
n (60)
(50)
c

Fig. 8.



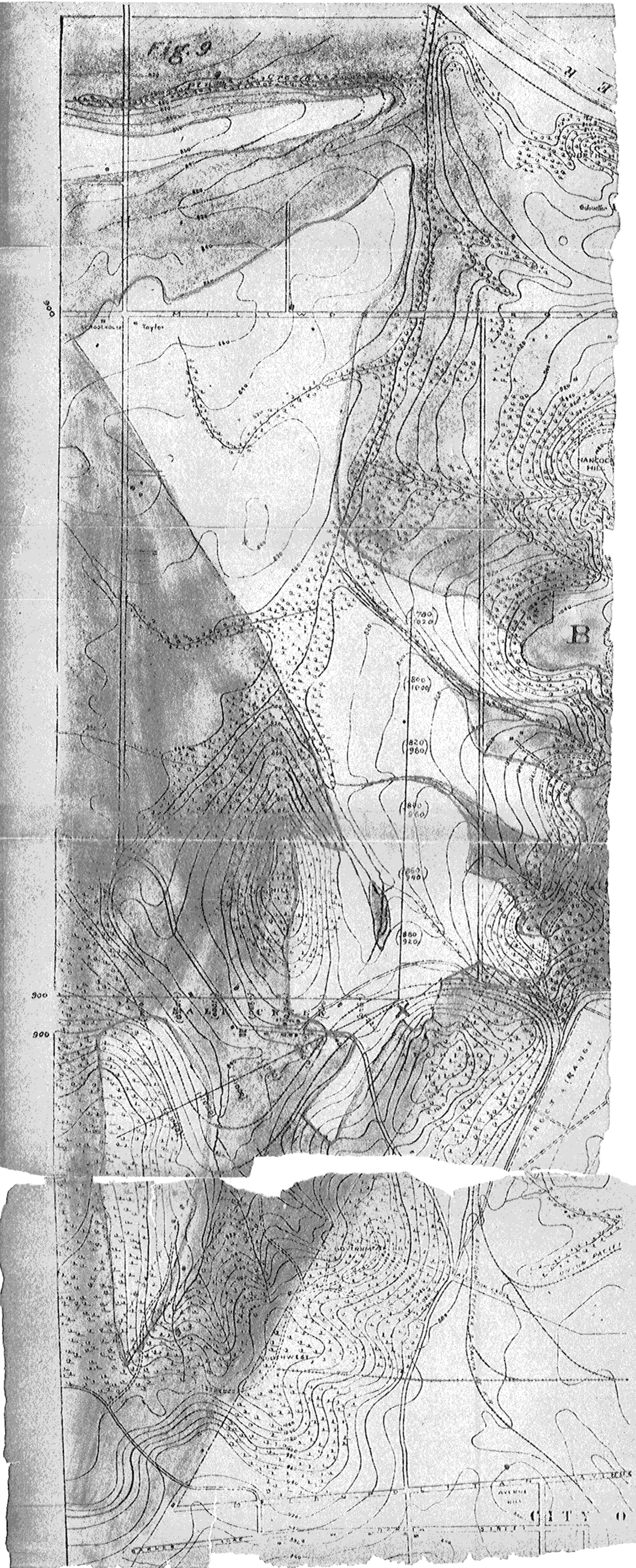


Fig. 9